FLUID STATICS

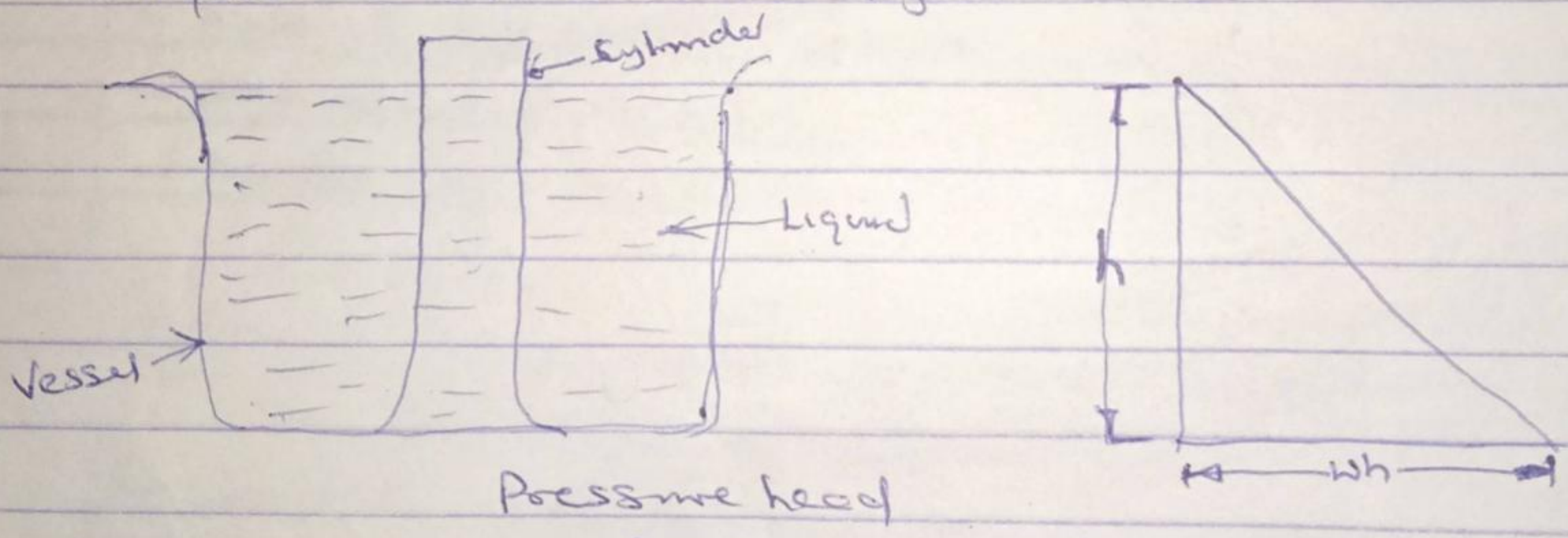
Pressure of a Liquid

When a fluid is contained in a vessel, it exerts force at all points on the sides and bottom and top of the container. The force per unit are is called Pressure. Pressure of a fluid on a surface always act normal to the surface.

Pressure head of a liquid

A liquid is subjected to pressure due to its own weight, this pressure increases as the depth of the liquid increases.

Considering a vessel containing liquid, as shown below. The liquid will exert pressure on all sides and bottom of the vessel. Let us consider cylinder made to stand in a liquid as shown in the figure



Pressure head

let h = Height of Liquid in the cylinder

A = Area of the cylinder base

w = Specific weight of the liquid

and P = Intensity of Pressure

Total Pressure on the base of cylinder = Weight of liquid in the cylinder

i.e $P.A = wAh$

$P = \frac{wAh}{A} = wh$ hence $P = wh$

As $p = wh$, the intensity of pressure in a liquid due to it depth will vary directly with depth

$$\therefore h = p/w$$

Hydrostatic Law states that the rate of increase of pressure in a vertically downward direction must be equal to the specific weight of the fluid at the point.

QUESTIONS - Find the height of water column corresponding to the pressure of 54 KN/m2

Solution

Given $P = 54\ KN/m^2$, $w = 9.81\ KN/m^3$, $h = ?$

using the relation $p = wh$

$$\therefore h = p/w = \frac{54}{9.81} = \underline{5.5\ m}$$

Assignment

- Pascal Law
- Absolute and gauges pressures
- Atmospheric pressure
- Devices used in measuring pressure.

Pressure measurement by manometer

The relationship between pressure and head is used for pressure measurement in the manometer or liquid gauge.

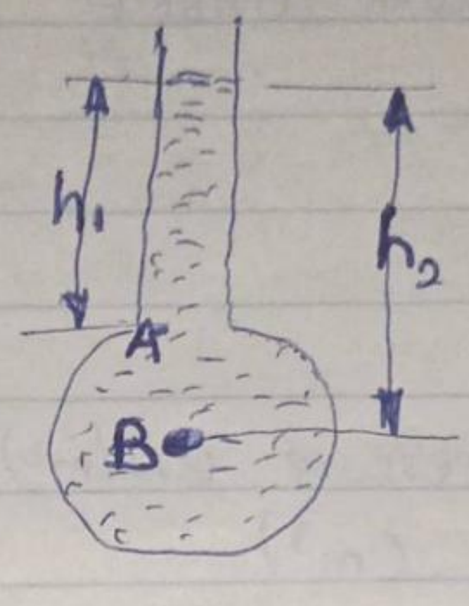
The simplest form manometer also called Piezometer tube It consist of a single vertical tube, open at the top, inserted into a pipe or vessel containing liquid under pressure which rises in the tube to a height depending on the pressure. If the top of the tube is open to the atmosphere, the pressure measured is (gauge) pressure

From the diagram below

Pressure at A = Pressure due to Column of liquid at height h,

$P_A = \rho g h_1$

Similarly $P_B = \rho g h_2$



~~Open End U-Tube Manometer~~

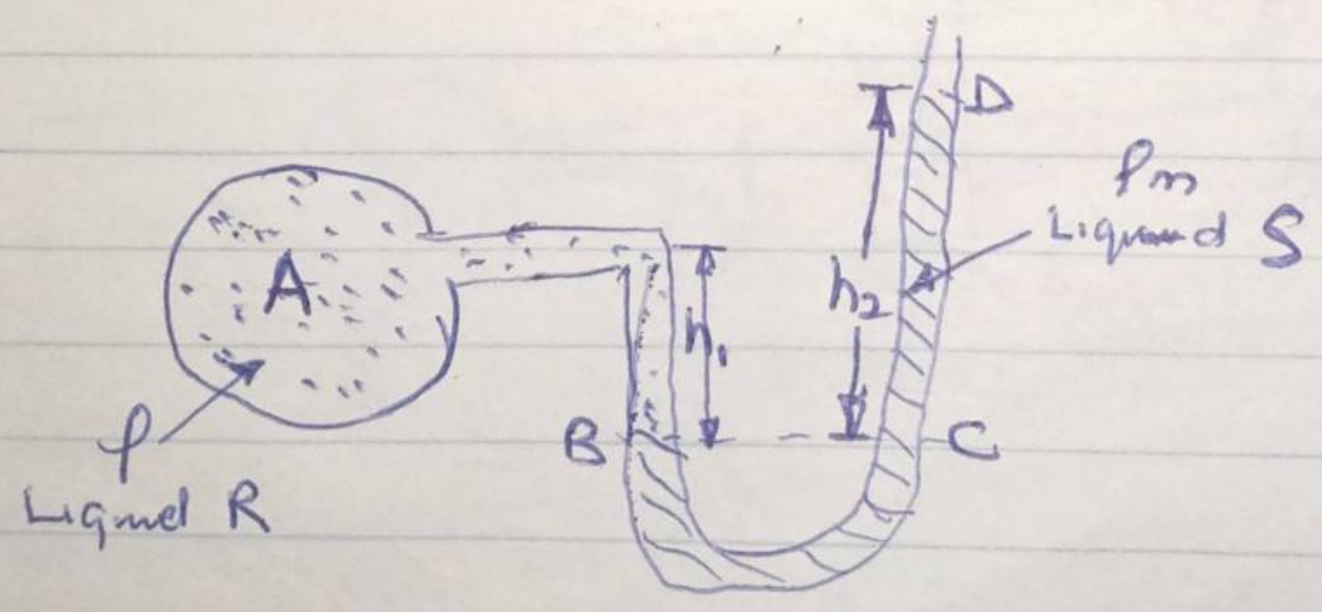
U-TUBE MANOMETER

Piezometers cannot be used when large pressures in the lighter liquids are to be measured, since this will required very long tubes, which cannot be handled conveniently. Furthermore gas pressures cannot be measured by the piezometers because a gas forms no free atmospheric surface. These limitations can be overcome by the use of U-tube manometers.

A U-tube manometer consists of a glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown below

Open-end U-tube manometer

This U-tube manometer can be used to measure the pressure of either liquid or gases.



The bottom of the U-tube is filled with a manometric liquid S which is of greater density ρ_m and is immiscible with the liquid R, it could be liquid or gas of density ρ, whose

Pressure is to be measured. If B is the level of the interface in the left-hand limb and C is a point at the same level in the right-hand limb.

Pressure P_B = Pressure P_C

but P_B = Pressure P_A + Pressure due to depth h_1 of fluid R

$= P_A + \rho g h_1$

P_C = Pressure P_D + Pressure due to depth h_2 of liquid S

but P_D = Atmospheric pressure = Zero gauge Pressure

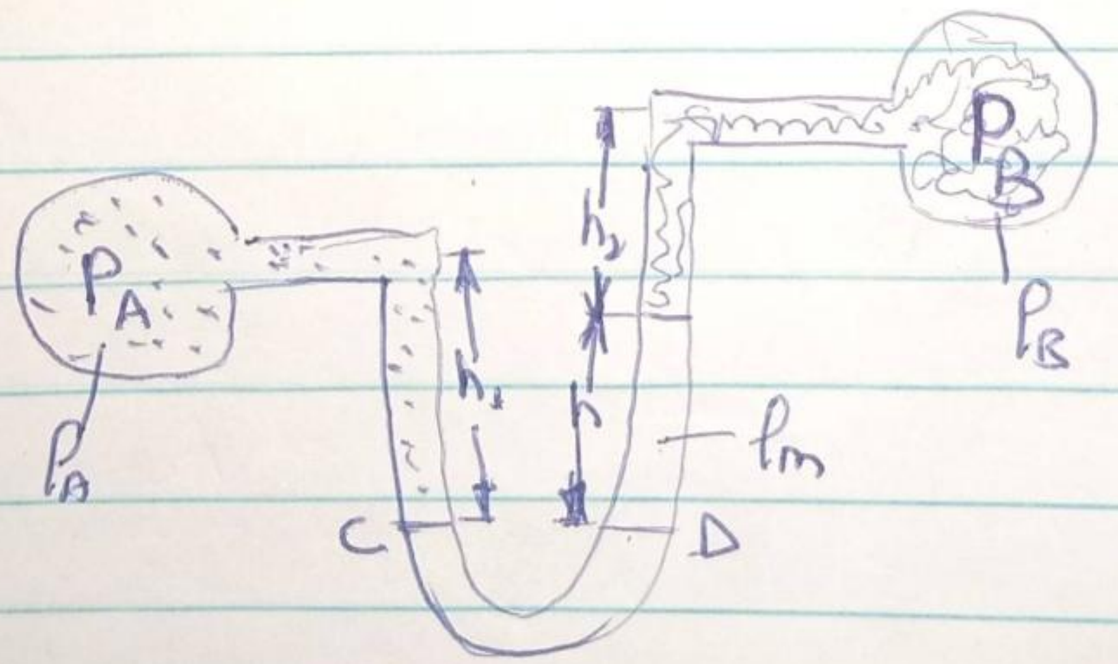
$P_C = 0 + \rho_m g h_2$ since $P_B = P_C$

$P_A + \rho g h_1 = \rho_m g h_2$

$P_A = \rho_m g h_2 - \rho g h_1$

Close-End U-Tube Manometer

The Pressure difference between two points in a fluid can be measured by measuring the pressure at each point separately and substracting or using a differential Pressure gauge or manometer which measures pressure difference between two different points in the same pipeline or two different chambers



The Principle involved in Calculating the pressure difference is that the pressure at the same level CD in the two limb must be equal, since the fluid in the bottom of the U-tube is at rest. Let P_A and P_B be the source pressu A and B the fluid ~~densities~~ densities, in the manometer

liquid densities and elevations h, h_1, and h_2 as shown
Equating the pressure in each limb at level CD

$P_c = P_A + \rho_A g h_1$ and

$P_D = P_B + \rho_B g h_2 + \rho_m g h$

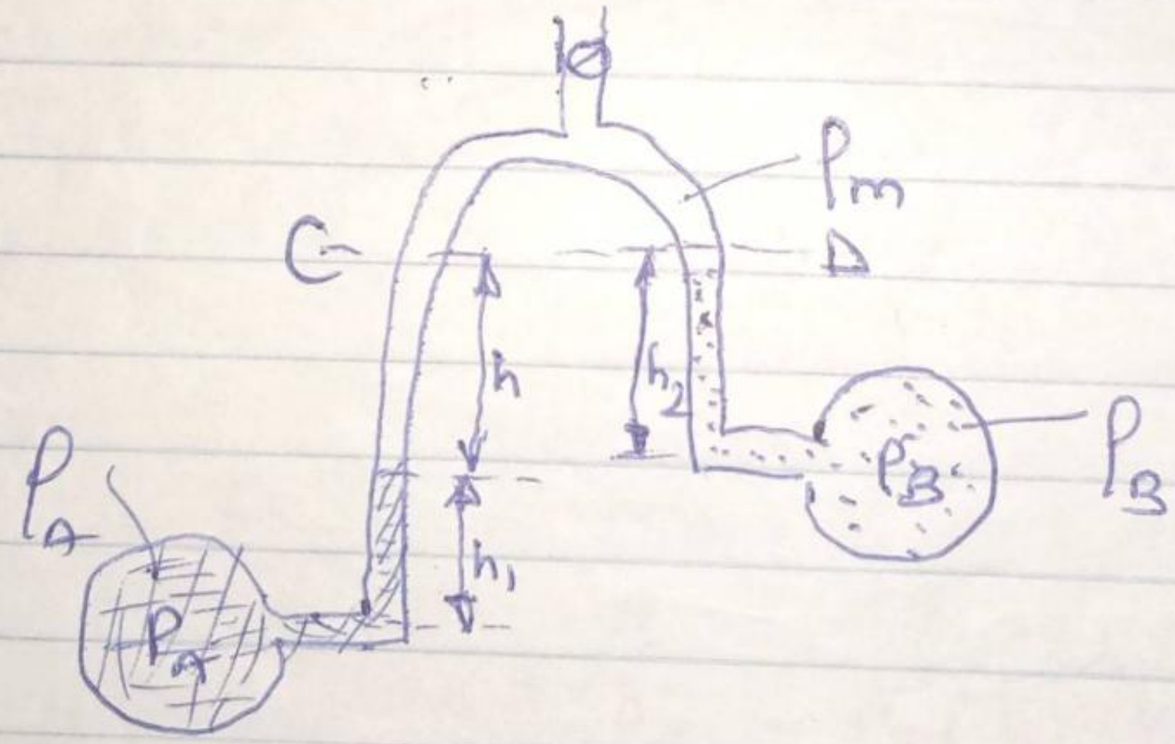
but $P_c = P_D$ hence

$P_A + \rho_A g h_1 = P_B + \rho_B g h_2 + \rho_m g h$

$P_A - P_B = \rho_B g h_2 + \rho_m g h - \rho_A g h_1$

Inverter U-Tube Manometer

The Inverter U-tube manometer is used for measuring pressure differences in liquids. The top of the U-tube is filled with fluid, frequently air, which is less dense than that connected to the instrument. Since the fluid at the top is at rest pressure at level CD will be the same in both limbs



$P_A = \rho_A g h_1 + \rho_m g h + P_c$ and $P_B = \rho_B g h_2 + P_D$

$P_A - P_B = \rho_A g h_1 + \rho_m g h + P_c - \rho_B g h_2 - P_D$

Since $P_c = P_D$

$P_A - P_B = \rho_A g h_1 + \rho_m g h - \rho_B g h_2$ ——— (1)

If the top of the tube is filled with air, ρ_m

will be negligible compared to P_A and P_B. Hence equation 1 becomes

$$P_A - P_B = \rho_A g h_1 - \rho_B g h_2 \quad -- \quad (2)$$

If fluid in A and B are the same or they are two different point in a pipeline conveying the same fluid, therefore $\rho_A = \rho_B = \rho$ and equation 1 becomes

$$P_A - P_B = \rho g(h_1 - h_2) + \rho_m g h$$

If the top of the tube is filled with air and fluid A and B are the same. Therefore equation 1 becomes

$$P_A - P_B = \rho g(h_1 - h_2)$$

QUESTIONS

An inverted U-tube as shown in the previous lecture is used to measure the pressure difference between two points A and B in a pipe line through which water is flowing. The top of the manometer is filled with oil of relative density 0.8, Calculate the pressure difference, if h_1 is 0.25m, h_2 is 0.45m and h is 0.3m

Solution

Given The fluid in A and B is water ~~$\rho = 1000$~~

$\therefore \rho = 1000 kg/m^3$, $\rho_m = 0.8 \times 1000 = 800 kg/m^3$

Pressure difference $P_A - P_B = \rho g(h_1 - h_2) + \rho_m g h$

$P_A - P_B = 1000 \times 9.8(0.25 - 0.45) + 800 \times 9.81 \times 0.3$

$= -1962 + 2354.4 = 392.4 N/m^2$